Lecture Notes
The Standard New-Keynesian Model

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9.1.2014

These notes are based on: Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. *DSGE Models for Monetary Policy Analysis.* Handbook of Monetary Economics. Elsevier, 2010.

1 A Small New Keynesian DSGE Model

1.1 Households

Households derive utility from consumption C_t and leisure $1 - H_t$ according to

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{H_t^{\phi+1}}{\phi+1} \right), \quad \beta \in (0,1), \phi \geq 0$$

Households can also lend and borrow by buying government bonds. Denoting the bond holding by B_t, their budget constraint is

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + W_t H_t + \Pi_t,$$

where P_t is the price of the final good, W_t is nominal wage, R_t is the nominal gross return on bonds, and Π_t is the sum of profits from owning firms and other transfers.

Denoting the lagrangian multiplier on the constraint as v_t, the households optimization problem leads to

$$C_t H_t^{\phi} = \frac{W_t}{P_t},$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1} C_{t+1}} \right],$$

$$v_t = \frac{1}{P_t C_t}.$$

where $\pi_{t+1} = P_{t+1}/P_t$.

1.2 Final Good Firms

The final good is produced by competitive firms from a measure one of intermediate goods using the technology

$$Y_t = \left(\int_0^1 Y_{it}^{1/\lambda} di \right)^{\lambda}, \ \lambda > 1. \tag{1}$$

The demand for the good Y_{it} is given by solving the representative firm's profit maximizing problem:

$$Y_{it} = Y_t \left(\frac{P_t}{P_{it}} \right)^{\lambda/(\lambda-1)}. \tag{2}$$

Using (2) in (1) gives us a relationship between the prices of the intermediate goods and the final good

$$P_t = \left(\int_0^1 P_{it}^{-1/(\lambda-1)} di \right)^{-(\lambda-1)}. \tag{3}$$

1.3 Intermediate Good Firms

The intermediate goods are each produced by a monopolistic firm using only labor. The technology is given by

$$Y_{it} = z_t H_{i,t},$$

where z_t is a stochastic process that we leave unspecified for now. The labor that the firm hires is subsidized by the government at a rate ν_t, so the real marginal cost of production is

$$s_t = (1 - \nu_t) \frac{W_t}{z_t P_t}.$$

If there were no price rigidities, the intermediate goods firm would all set $P_{it} = \lambda P_t s_t$, i.e. a constant markup above the marginal cost.

A key assumption in the New Keynesian model is that there are price setting rigidities. We assume that at each period each intermediate good firm is allowed to set its price at probability $1 - \xi$ and is otherwise forced to keep the price from the previous period: $P_{i,t} = P_{i,t-1}$.

Before we solve the intermediate good firm's problem, first notice that all firms that get to update their prices at period t are facing the exact same problem (since z_t is common to all firms), so they will choose the same price, which we denote

$\tilde{P}_t$. Furthermore, since the firms that don't get to update at time t are chosen at random, the price distribution of those firms at time t is identical to the overall price distribution at time $t-1$. Using (3) this implies that

$$P_t^{-1/(\lambda-1)} = \int_0^1 \left[\xi P_{i,t-1}^{-1/(\lambda-1)} + (1-\xi)\tilde{P}_t^{-1/(\lambda-1)}\right] di =$$

$$= \xi \int_0^1 P_{i,t-1}^{-1/(\lambda-1)} di + (1-\xi)\tilde{P}_t = \xi P_{t-1}^{-1/(\lambda-1)} + (1-\xi)\tilde{P}_t^{-1/(\lambda-1)}.$$

Therefore

$$\tilde{p}_t^{-1/(\lambda-1)} = \frac{1-\xi\pi_t^{1/(\lambda-1)}}{1-\xi}, \tag{4}$$

where $\tilde{p}_t = \tilde{P}_t/P_t$.

To solve the firm's problem, note that since there is now a dynamic aspect to the firm's behavior, the firm does not simply maximize its period profit, but takes into account the possibility that it will not be able to change its prices in the future. The firm's problem is therefore

$$\max_{\tilde{P}_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\xi)^\tau v_{t+\tau}(\tilde{P}_t - P_{t+\tau}s_{t+\tau})Y_{i,t+\tau}. \tag{5}$$

The ξ^τ factor is the probability that the firm will not be allowed to update its price for the next $t+\tau$ periods. The weight $\beta^t v_t$ in the sum is the Lagrange multiplier on the household's budget constraint. The reason the firm uses this weight is that firms are owned by the households, and the multiplier, interpreted as a shadow price, represents the benefit for the household from extra income at that period.

Solving the maximization problem for the firm leads to

$$\tilde{p}_t = \frac{\lambda \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\xi)^\tau X_{t,\tau}^{-\lambda/(\lambda-1)} s_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\xi)^\tau X_{t,\tau}^{-1/(\lambda-1)}} = \frac{K_t^f}{F_t^f}, \tag{6}$$

where K_t^f and F_t^f are the numerator and denominator of the ratio after the first equality, and

$$X_{t,\tau} = \begin{cases} 1 & \tau = 0, \\ [\pi_{t+1}\pi_{t+2}\cdots\pi_{t+\tau}]^{-1} & \tau > 0. \end{cases}$$

We can write recursive expressions for K_t^f and F_t^f:

$$K_t^f = \lambda s_t + \beta\xi \mathbb{E}_t \pi_{t+1}^{\lambda/(\lambda-1)} K_{t+1}^f \tag{7}$$

$$F_t^f = 1 + \beta\xi \mathbb{E}_t \pi_{t+1}^{1/(\lambda-1)} F_{t+1}^f. \tag{8}$$

1.4 Price Dispersion

There is no aggregate production function in the New Keynesian model production, i.e. knowing the factors of production does not uniquely determine output. Since some firms are not allowed to update their price, labor is allocated inefficiently and the overall loss of output depends on the exact distribution of the prices of the intermediate goods. To measure the output loss, define

$$Y_t^* = \int_0^1 Y_{it} di = \int_0^1 z_t H_{it} di = z_t H_t.$$

On the other hand, using (2)

$$Y_t^* = \int_0^1 Y_{it} di = \int_0^1 Y_t \left(\frac{P_t}{P_{it}}\right)^{\lambda/(\lambda-1)} di = Y_t \left(\frac{P_t}{P_t^*}\right)^{\lambda/(\lambda-1)},$$

where

$$P_t^* = \left(\int_0^1 P_{it}^{-\lambda/(\lambda-1)} di\right)^{-(\lambda-1)/\lambda}.$$

We can now write an aggregate expression for output

$$Y_t = p_t^* z_t H_t, \quad \text{where} \quad p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\lambda/(\lambda-1)}. \tag{9}$$

In the appendix, we prove that $p_t^* \leq 1$ using Jensen's inequality. This also proves that output is generally inefficient, since allocating labor equally between all firms would lead to $Y_t = z_t H_t$. In fact, p_t^* can be thought of as a measure of the inefficiency.

Once again, the assumption that the firms that are allowed to set prices are chosen uniformly leads to an expression similar to (4)

$$\begin{aligned} P_t^{*-\lambda/(\lambda-1)} = \int_0^1 P_{it}^{-\lambda/(\lambda-1)} di = \int_0^1 \left[\xi P_{i,t-1}^{-\lambda/(\lambda-1)} + (1-\xi)\tilde{P}_t^{-\lambda/(\lambda-1)}\right] di = \\ = \xi {P_{t-1}^*}^{-\lambda/(\lambda-1)} + (1-\xi)\tilde{P}_t^{-\lambda/(\lambda-1)}. \end{aligned}$$

Therefore,

$$p_t^{*-1} = \xi \pi_t^{\lambda/(\lambda-1)} {p_{t-1}^*}^{-1} + (1-\xi)\tilde{p}_t^{-\lambda/(\lambda-1)}.$$

Using (4) to substitute out $\tilde{p}_t$,

$$p_t^* = \left[\xi \frac{\pi_t^{\lambda/(\lambda-1)}}{p_{t-1}^*} + (1-\xi)\left(\frac{1-\xi\pi_t^{1/(\lambda-1)}}{1-\xi}\right)^{\lambda}\right]^{-1} \tag{10}$$

1.5 Private Sector Equilibrium

We can summarize the economy by the following ten unknowns:

$$C_t, H_t, R_t, \pi_t, p_t^*, K_t^f, F_t^f, \frac{W_t}{P_t}, s_t, \nu_t.$$

The equations we have so far for these are:

$$C_t H_t^{\phi} = \frac{W_t}{P_t}, \tag{11a}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1} C_{t+1}} \right], \tag{11b}$$

$$s_t = (1 - \nu_t) \frac{W_t}{z_t P_t}, \tag{11c}$$

$$K_t^f = \lambda s_t + \beta \xi \mathbb{E}_t \pi_{t+1}^{\lambda/(\lambda-1)} K_{t+1}^f, \tag{11d}$$

$$F_t^f = 1 + \beta \xi \mathbb{E}_t \pi_{t+1}^{1/(\lambda-1)} F_{t+1}^f, \tag{11e}$$

$$K_t^f = F_t^f \left[\frac{1 - \xi \pi_t^{1/(\lambda-1)}}{1 - \xi} \right]^{-(\lambda-1)}, \tag{11f}$$

$$p_t^* = \left[\xi \frac{\pi_t^{\lambda/(\lambda-1)}}{p_{t-1}^*} + (1 - \xi) \left(\frac{1 - \xi \pi_t^{1/(\lambda-1)}}{1 - \xi} \right)^{\lambda} \right]^{-1}, \tag{11g}$$

$$C_t = p_t^* z_t H_t. \tag{11h}$$

All of these equations are copied from above with the exception of (11f), which is simply (4) with $\tilde{p}_t = K_t^f / F_t^f$; and (11h), which is (9) together with the market clearing condition $Y_t = C_t$.

Equations (11) define the private sector equilibrium. These are eight equations for ten unknowns. One equation we are missing is defining the subsidy ν_t. We have seen that without the price rigidities, the intermediate good firms set prices at a fixed markup above marginal cost $P_{it} = \lambda P_t s_t$, so one standard approach, which we follow here, is to set the subsidy to be constant at $\nu_t = 1 - 1/\lambda$. This assures that at the steady-state production is efficient.

The other equation we are missing is for monetary policy. We will assume that monetary policy is implemented via a Taylor rule for interest rate, but we postpone this discussion till after we linearize the model.

1.6 The Centralized Model

Before we continue, let's consider how a central planner would optimize the allocations in this economy. Obviously, the planner would simply allocate labor equally between the intermediate good firms, so that for all i, $Y_{it} = z_t H_t$ and therefore $Y_t = z_t H_t$. We can therefore write the planner's problem as

$$\max_{\{C_t, H_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{H_t^{1+\phi}}{1+\phi} \right], \text{ s.t.} \quad C_t = z_t H_t.$$

Defining $x_t = C_t / z_t$, we can rewrite this as

$$\max_{\{x_t, H_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log x_t - \frac{H_t^{1+\phi}}{1+\phi} \right], \text{ s.t.} \quad x_t = H_t.$$

Written in this way, this problem actually has no state variables, so it is obvious that the planner would choose x_t and H_t to be constant over time. This implies that the optimal C_t is proportional to z_t.

In the New-Keynesian model, the deviation of x_t from its steady state value is called the *output gap*, since it is precisely the difference between the potential and the actual output.

1.7 The Linearized NK Model

The main characteristics of the New Keynesian model are easier to see from the linearized equations, so we start by linearizing the model around a steady-state solution.

Obviously π_t fluctuates around its steady state value, so equation (11g) implies that p_t^* must also fluctuate. However, the changes in p_t^* vanish to first order approximation since at steady-state $p^* = 1$, and this is also a local maximum value for p_t^*. This can also be seen by linearizing (11g) around $p^* = \pi = 1$:

$$\hat{p}_t^* = \xi \hat{p}_{t-1}^*,$$

where for any variable a hat indicates its relative deviation from its steady-state value: $\hat{\omega} = (\omega - \omega^{SS})/\omega^{SS}$. We see that if we set $\hat{p}_0^* = 0$, it will remain at this value for all t.

Linearizing equations (11d),(11e),(11f) around the steady-state

$$\hat{K}_t^f = (1 - \beta\xi)\hat{s}_t + \beta\xi \mathbb{E}_t \left[\frac{\lambda}{\lambda - 1} \hat{\pi}_{t+1} + \hat{K}_{t+1}^f \right], \tag{12a}$$

$$\hat{F}_t^f = \beta\xi \mathbb{E}_t \left[\frac{1}{\lambda - 1} \hat{\pi}_{t+1} + \hat{F}_{t+1}^f \right], \tag{12b}$$

$$\hat{K}_t^f = \hat{F}_t^f + \frac{\xi}{1 - \xi} \hat{\pi}_t. \tag{12c}$$

Subtract equation (12b) from (12a) and use (12c) both for t and $t+1$,

$$\hat{K}_t^f - \hat{F}_t^f = (1-\beta\xi)\hat{s}_t + \beta\xi\mathbb{E}_t\left[\hat{\pi}_{t+1} + \hat{K}_{t+1}^f - \hat{F}_{t+1}^f\right],$$
$$\hat{\pi}_t = \frac{(1-\xi)(1-\beta\xi)}{\xi}\hat{s}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{13}$$

Linearizing (11a), (11c) and (11h)

$$\hat{C}_t + \phi\hat{H}_t = \hat{w}_t, \tag{14}$$
$$\hat{s}_t = \hat{w}_t - \hat{z}_t, \tag{15}$$
$$\hat{C}_t = \hat{z}_t + \hat{H}_t, \tag{16}$$

where $w_t = W_t/P_t$ is the real wage. Eliminating $\hat{w}_t$ and z_t

$$\hat{s}_t = (1+\phi)\hat{H}_t = (1+\phi)(\hat{C}_t - \hat{z}_t) = (1+\phi)\hat{x}_t.$$

Recall that $x_t = C_t/z_t$. Using this in (13) gives

$$\boxed{\hat{\pi}_t = \kappa\hat{x}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1},} \tag{17}$$

where

$$\kappa = \frac{(1-\xi)(1-\beta\xi)}{\xi}(1+\phi).$$

Equation (17) is the celebrated new-Keynesian Phillips curve.

Linearizing (11b)

$$\mathbb{E}_t\left[\hat{C}_t - \hat{C}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1}\right] = 0.$$

Define $\hat{R}_t^* = \mathbb{E}_t[\hat{z}_{t+1} - \hat{z}_t]$, and rewrite

$$\mathbb{E}_t\left[\hat{x}_t - \hat{x}_{t+1} - \hat{R}_t^* + \hat{R}_t - \hat{\pi}_{t+1}\right] = 0.$$

or

$$\boxed{\hat{x}_t = \mathbb{E}_t\left[\hat{x}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^*)\right].} \tag{18}$$

Equation (18) is the new-Keynesian IS curve.

1.8 The Taylor Rule

The linearized new-Keynesian model reduces to to three variables: $(\hat{x}_t, \hat{\pi}_t, \hat{R}_t)$, for which we have two equations: the Phillips curve (17) and the IS curve (18). The third equation defines the monetary policy. Conceptually, what a benevolent government should do in this environment is choose a rule $\hat{R}_t(\hat{x}_t, \hat{\pi}_t)$ that maximizes the welfare of the households. Unfortunately, this problem is beyond the scope of this course, so instead we simply postulate that the monetary authority is following a Taylor rule:

$$\hat{R}_t = r_\pi \mathbb{E}_t \hat{\pi}_{t+1} + r_x \hat{x}_t. \tag{19}$$

From here on I drop the "hats" over $\hat{x}_t, \hat{\pi}_t$ and $\hat{R}_t$, but they are implied.

The three equations (17), (18), and (19) for the three variables (x_t, π_t, R_t) are known as the 3-equation New Keynesian model. The 3-equation model, despite its simplicity, gives many insights about the nature of the business cycle in NK approach. For example, we can see how the Taylor Principle, the policy recommendation to set $r_\pi > 1$ comes about.

Before showing this formally, we can gain some intuition by starting with the Phillips curve (17)

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1},$$

and iterating forward in time

$$\begin{aligned}\pi_t &= \kappa \mathbb{E}_t[x_t + \beta x_{t+1}] + \beta^2 \mathbb{E}_t \pi_{t+2} \\ &= \kappa \mathbb{E}_t[x_t + \beta x_{t+1} + \beta^2 x_{t+2}] + \beta^3 \mathbb{E}_t \pi_{t+3} \\ &= \cdots = \kappa \sum_{\tau=0}^{\infty} \beta^\tau \mathbb{E}_t x_{t+\tau} + \lim_{\tau\to\infty} \beta^\tau \mathbb{E}_t \pi_{t+\tau} = \kappa \sum_{\tau=0}^{\infty} \beta^\tau \mathbb{E}_t x_{t+\tau}. \end{aligned} \tag{20}$$

At the last step we have assumes a non-explosive path for inflation. Similarly, from the IS curve (18)

$$\begin{aligned} x_t &= \mathbb{E}_t[x_{t+1} - (R_t - \pi_{t+1} - R_t^*)] \\ &= -\mathbb{E}_t[R_t - \pi_{t+1} - R_t^*] + \mathbb{E}_t[x_{t+2} - (R_{t+1} - \pi_{t+2} - R_{t+1}^*)] \\ &= -\mathbb{E}_t \sum_{\tau=0}^{\infty} [R_{t+\tau} - \pi_{t+\tau+1} - R_{\tau+1}^*] + \lim_{\tau\to\infty} \mathbb{E}_t x_{t+\tau}. \end{aligned}$$

What we see is that the current inflation is a function of the present and future output gap, and that the current output gap depends on the long term real interest rate.

If we substituting the Taylor rule into the IS curve (18), and only then iterate forward

$$\begin{aligned}
x_t =& -(1+r_x)^{-1}\mathbb{E}_t[(r_\pi - 1)\pi_{t+1} - R_t^*] + (1+r_x)^{-1}\mathbb{E}_t[x_{t+1}] \\
=& -(1+r_x)^{-1}\mathbb{E}_t[(r_\pi - 1)\pi_{t+1} - R_t^*] \\
& -(1+r_x)^{-2}\mathbb{E}_t[(r_\pi - 1)\pi_{t+2} - R_{t+1}^*] + (1+r_x)^{-2}\mathbb{E}_t[x_{t+2}] \\
=& \cdots \\
=& -\sum_{\tau=0}^{\infty}(1+r_x)^{-\tau-1}\mathbb{E}_t[(r_\pi - 1)\pi_{t+\tau+1} - R_{t+\tau}^*] + \lim_{\tau\to\infty}(1+r_x)^{-\tau}\mathbb{E}_t[x_{t+\tau}] \\
=& -\sum_{\tau=0}^{\infty}(1+r_x)^{-\tau-1}\mathbb{E}_t[(r_\pi - 1)\pi_{t+\tau+1} - R_{t+\tau}^*].
\end{aligned}$$

Now suppose that the economy develops expectations for positive inflation at some future date. The last equation tells us that if $r_\pi < 1$, then this will cause a positive output gap, and (20) means that this will cause current inflation, which again causes more output gap, etc.. If instead $r_\pi > 1$, then the expectations lead to a negative output gap, which decreases current inflation, so that the 'wrong' expectations quickly dissipate. The mechanism at work is that inflation expectations lead to a rise in the interest rate, which reduces spending, and with less resources used, reduces marginal costs. The lower marginal costs lead to lower actual inflation. Of course, in a rational expectations equilibrium the wrong expectations about future inflation would never develop to being with, but this analysis also works in models that instead assume that agents must learn the rules of the economy.

To prove this more formally, write the system in matrix notation

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -\beta & 0 \\ 0 & -r_\pi & 0 \end{pmatrix} \mathbb{E}_t \begin{pmatrix} x_{t+1} \\ \pi_{t+1} \\ R_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ \kappa & -1 & 0 \\ r_x & 0 & -1 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \\ R_t \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \mathbb{E}_t R_t^*.$$

The matrix on the left is not invertible, so we must reduce the system to two dimensions, after which we find the two eigenvalues $\lambda_{1,2}$ and the corresponding eigenvectors $v_{1,2}$. The general solution is

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = a_1 v_1 \lambda_2^t + a_1 v_2 \lambda_2^t + v_{nh},$$

where v_{nh} is the nonhomogeneous part. Since both x_t and π_t are choice variables in the system, the only way to get uniqueness is if both eigenvalues are larger than unity, since this will force us two set $a_1 = a_2 = 0$. It can be shown that $r_\pi > 1$ is a sufficient condition for that.

2 A Medium New Keynesian DSGE Model

The model in the previous section is obviously insufficient for most macroeconomic analysis since it lacks capital, a fiscal sector, and so on. There has been a great amount of work in the past quarter century on making changes to DSGE to improve their accuracy.

At the end of these notes I attach impulse response graphs for one such model.[1] To get a sense of the model, I simply list some of its features here:

- The households' period utility from consumption is given by $u(c_t - bc_{t-1})$. $b > 0$ is a habit formation parameter, and it makes households more averse to changing their consumption level. This is useful for making consumption less volatile in the model, as is empirically observed.
- Intermediate good firms produce using capital and labor.
- Both prices and wages are sticky *à la* Calvo, but firms and workers are allowed to index, that is, a firm that doesn't get to update its price can have it increase by a predetermined ratio.
- There is a distinction between the capital stock and the amount of capital utilized. Installing capital is costly.

A Proof that $p_t^* \leq 1$ using Jensen's Inequality

Jensen's inequality states that for any non-negative function $f : [a, b] \to \mathbb{R}$, and φ a convex function,

$$\varphi\left[\int_a^b f(i)di\right] \leq \frac{1}{b-a}\int_a^b \varphi((b-a)f(i))di.$$

Let $[a, b] = [0, 1]$, $f(i) = P_{it}^{-1/(\lambda-1)}$ and $\phi(x) = x^\lambda$ (which is convex since $\lambda > 1$). Then according to Jensen's inequality

$$\left[\int_0^1 P_{it}^{-1/(\lambda-1)}di\right]^\lambda \leq \int_0^1 P_{it}^{-\lambda/(\lambda-1)}di$$

$$\left[P_t^{-1/(\lambda-1)}\right]^\lambda \leq P_t^{*-\lambda/(\lambda-1)}$$

$$P_t^{-\lambda/(\lambda-1)} \leq P_t^{*-\lambda/(\lambda-1)} \qquad \Rightarrow \qquad p_t^* = \left(\frac{P_t^*}{P_t}\right)^{\lambda/(\lambda-1)} \leq 1.$$

[1] Taken from Christiano, Eichenbaum, and Evans, *DSGE Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,* Journal of Political Economy, Vol. 113, No. 1, 2005, pp 1-45.

as needed.

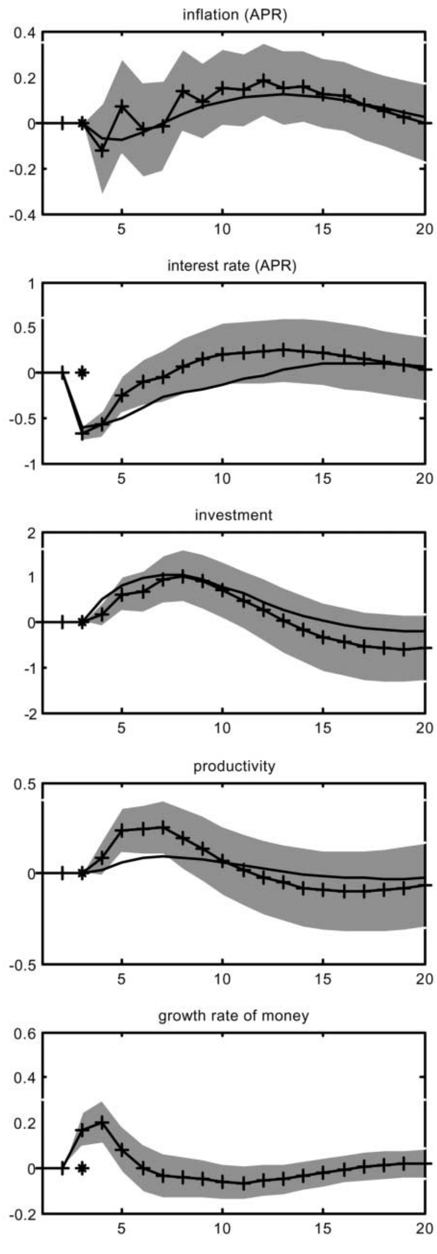

Fig. 1.—Model- and VAR-based impulse responses. Solid lines are benchmark model impulse responses; solid lines with plus signs are VAR-based impulse responses. Grey areas are 95 percent confidence intervals about VAR-based estimates. Units on the horizontal axis are quarters. An asterisk indicates the period of policy shock. The vertical axis units are deviations from the unshocked path. Inflation, money growth, and the interest rate are given in annualized percentage points (APR); other variables are given in percentages.

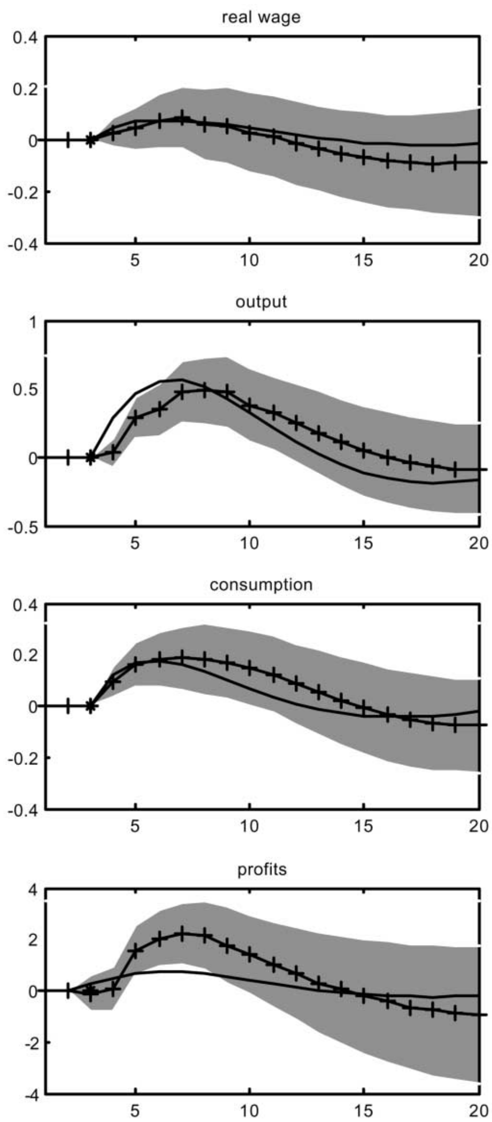

FIG. 1.—*Continued*